TRANSPORTATION AND LOCATION-ALLOCATION PROBLEMS

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Abstract: This article provides a comprehensive overview of transportation and location allocation problems within transportation and logistics systems theory, with emphasis on their mathematical formulations. Recent research developments, including optimization approaches and real-world applications, are discussed. Two case studies from the Slovak automotive sector highlight practical implementations, specifically plant location and inbound logistics optimization.
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Introduction

Transportation and logistics systems theory provides the basis for planning how goods are moved and where facilities are located in supply chains. These decisions are particularly critical in regions with high levels of logistics activity. For example, Slovakia's central location in Europe makes it an ideal regional hub for transportation and logistics networks [1]. The country has developed a strong automotive cluster with four major car assembly plants (Volkswagen in Bratislava, Stellantis-PSA in Trnava, Kia in Žilina, and Jaguar Land Rover in Nitra) [2]. The automotive sector accounts for almost half of Slovakia's industrial production and exports [2]. In 2019, the country produced over 1.1 million vehicles - approximately 202 cars per 1,000 people - the highest per capita automotive production in the world [2][3]. Efficient transportation and facility location strategies are therefore paramount to maintaining this competitive advantage.

In this context, the theory of transportation and logistics systems focuses on optimization problems that ensure the cost-effective and timely delivery of goods. Two fundamental classes of problems in this theory are transportation problems (optimizing shipment flows between supply and demand points) and location allocation problems (choosing optimal locations for facilities and assigning customers to them). This article provides an overview of these problems. In the introduction, the importance of such problems in the real world is outlined. Next, an overview of the current state of research is given, highlighting recent developments. Finally, the theoretical foundations of transport and location assignment problems are discussed, including mathematical formulations and examples illustrating their application in the Slovak automotive logistics sector.

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Overview of current research

Transportation problems: The transportation problem is a classic optimization model in logistics. It was originally formulated in the 1940s (also known as the Hitchcock problem) and has since become a standard example of linear programming in supply chain optimization [4]. Because it can be solved efficiently by linear programming methods, the transportation problem is considered essentially "solved" from an algorithmic point of view. Research in this area today often focuses on extending the basic model to more complex scenarios or integrating it with other decision problems. For example, recent studies consider multimodal transportation (e.g., combining road and rail) and real-time routing, reflecting industry trends toward flexible and sustainable logistics. In Slovakia, where about 50% of finished vehicles are transported by rail in multimodal networks [1], researchers and practitioners are interested in models that incorporate such modalities and constraints (e.g., schedules, capacities) into transportation optimization. Another active area of research is the location-transportation problem, which combines facility location decisions with flow decisions. This integrated approach recognizes that the optimal transportation plan may depend on facility locations and vice versa. Uncertainty and robustness are also studied: for example, robust transportation models account for demand fluctuations or disruptions, and aim to find solutions that minimize costs while meeting service level objectives under different scenarios.

Location-Allocation Problems: Location-Allocation (L-A) problems remain a vibrant area of research due to their complexity and impact on strategic logistics planning. These problems are generally NP-hard, meaning that exact solutions become computationally infeasible for large instances [5]. As a result, there is a rich literature on advanced algorithms for solving L-A problems. Researchers have developed both exact optimization methods (e.g., branch-and-bound, cutting planes) for moderately large instances and heuristic or metaheuristic approaches (genetic algorithms, tabu search, etc.) for large-scale or real-time applications. There are four main classical models in the area of location assignment: the p-Median, the Simple Plant Location Problem (also known as the Uncapacitated Plant Location Problem), the p-Center, and Covering models. Each addresses a different objective-for example, the p-median model minimizes the total distance or cost to all customers, while the p-center model minimizes the maximum distance each customer must travel to the nearest facility. Recent research has extended these models to address new objectives and constraints. One prominent trend is the incorporation of sustainability and resilience into location decisions. For example, "green" location models include carbon emissions in the objective function or constrain the environmental impact of logistics networks. In practice, this is consistent with efforts by automotive companies to reduce their carbon footprint by designing networks that minimize total transportation distance or favor lower-emission modes. Another current focus is on dynamic and stochastic location problems - where location decisions can be adjusted over time or must be robust to uncertain demand.

The rise of e-commerce and new delivery technologies has also spurred specialized L-A research, such as the optimal placement of urban distribution hubs or drone delivery stations. Overall, the field is moving toward more integrated and realistic models, solved with increasingly sophisticated algorithms and often aided by geographic information systems (GIS) and big data analytics for calibration [4].

In Slovakia, academic and industrial researchers are following these global trends. For example, Kádárová et al. (2021) propose optimization models for automotive supplier networks that balance risk and just-in-time performance [2]. Other studies focus on optimizing logistics infrastructure to support the growth of the country's automotive sector, such as evaluating potential locations for new logistics parks or improving the capacity of intermodal terminals in response to growing rail traffic. These efforts illustrate how current research in transportation and logistics systems is being directly applied to support real-world logistics challenges in the Slovak automotive industry.

Theoretical Foundations

Transportation Problem

The transportation problem is a fundamental model in logistics and operations research that deals with the optimal distribution of goods from multiple supply points to multiple demand points. In the basic scenario, we have a set of supply nodes (e.g., factories or warehouses) with given supply quantities and a set of demand nodes (e.g., customer locations or regional distribution centers) with desired demand quantities. Shipping costs are defined for moving a unit of product from each supply node to each demand node. The objective is to determine how much to ship along each route (supply node to demand node) so that all demands are satisfied without exceeding supply at minimum total cost.

Mathematically, the classical transportation problem can be formulated as a linear program. Let *I* be the index set of supply nodes and *J* the index set of demand nodes. Let a_i be the supply available at source $i \in I$, b_j the demand required at destination $j \in J$, c_{jj} the unit transportation cost from *i* to *j*. The decision variable x_{ij} represents the quantity transported from *i* to *j*. The formulation of the linear programming is:

$$\min_{\boldsymbol{x_{ij}}} \sum_{\boldsymbol{i \in I}} \sum_{\boldsymbol{j \in J}} \boldsymbol{c_{ij}\, x_{ij}}$$

$$\sum_{j \in J} x_{ij} = a_{i,} \qquad \forall i \in I \ \ (supply\ constraints),$$

$$\sum_{i \in I} x_{ij} = b_{j,} \qquad \forall j \in J \ \ (demand\ constraints),$$

$$x_{ij} \geq 0, \quad \forall i, j.$$

In essence, the first set of constraints ensures each supply node sends out no more than it produces, and the second set ensures each demand node receives exactly the amount it needs. If total supply equals total demand (a *balanced* transportation problem), all constraints can be satisfied exactly; if not, a dummy supply or demand node can be introduced to balance the equation.

The transportation problem is a special case of a minimum-cost network flow problem on a bipartite network (sources connected directly to destinations). This structure allows very efficient solution. The simplex algorithm can solve even large transportation problems relatively quickly, and specialized algorithms (such as the *transportation simplex* or the *stepping-stone method*) exploit the network structure for faster computations. In fact, the constraint matrix of this linear program is totally unimodular, which guarantees that an optimal basic solution will be integer-valued (so shipments can be treated as indivisible units without needing a separate integer programming solver).

Example (Automotive Inbound Logistics): In Slovakia's automotive industry, the transportation problem framework is used to optimize inbound logistics. A car assembly plant sources thousands of parts from various suppliers. This scenario can be modeled with suppliers as the "sources" and the assembly plant as a single "sink" (demand point) that needs specific quantities of each part. Logistics providers (third-party integrators) often consolidate shipments from multiple suppliers by setting up a network of collection points (cross-docks). For instance,

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an integrator might operate a cross-dock near Žilina where suppliers from around the region deliver their components; these are then consolidated into full truckloads bound for the Kia Motors assembly plant. By solving a transportation problem, the integrator can determine the cost-minimizing allocation of each supplier's shipments to different trucks or routes, while ensuring the plant's demand for each part is met. Furthermore, they employ strategies like milk-run deliveries – a single truck picking up from multiple suppliers in sequence – which is essentially a routing optimization built on top of the transportation plan. The milk-run ensures high vehicle utilization (trucks never run empty on their circuit) [8], reducing the total transportation cost per unit. The use of such methods helped Slovak automakers maintain efficient operations; even during the COVID-19 disruptions, flexible transport planning was crucial to handle supply interruptions [2].

Location-Allocation Problems

Location-allocation problems address the question of where to establish facilities (location decisions) and how to assign demand points to those facilities (allocation decisions) in order to optimize one or more objectives. These objectives can include minimizing transportation costs, maximizing coverage of clients, minimizing response time, or balancing workloads among facilities, depending on the context. Typical facilities considered in these problems include warehouses, distribution centers, factories, retail outlets, or service centers (like hospitals or fire stations), and demand points are usually customer locations or markets.
Several classical models fall under the location-allocation umbrella [5]:

- **p-Median Problem:** Select p facility sites out of a given set such that the total distance or transportation cost from each demand point to its nearest open facility is minimized. This model tends to locate facilities in a way that reduces the average distance customers travel.
- **Uncapacitated Facility Location Problem (UFLP):** Decide which facility sites to open (from a set of candidates) and assign each demand point to an open facility. The objective is to minimize the sum of fixed facility opening costs and transportation costs for serving all demands. This model (also called the simple plant location problem) generalizes the p-median by including facility fixed costs and not fixing the number of facilities in advance.
- **p-Center Problem:** Choose p facilities to minimize the maximum distance any demand point must travel to a facility. This "minimax" model is critical for emergency services, where the worst-case response time needs to be as small as possible.
- **Covering Problems:** Ensure that all demand points are within a certain distance or time threshold of a facility. Variants include maximizing the number of demands covered by p facilities (maximal covering) or minimizing the number of facilities needed to cover all demand (set covering).

Most location-allocation models can be formulated as mixed-integer programming problems. As an example, consider the uncapacitated facility location problem described above. Let J be the set of potential facility locations and I the set of demand points. For each location $j \in J$, there is a fixed cost f_j to open a facility there. For each demand point $i \in I$, if it is served by facility j, a transportation cost c_{ij} is incurred (e.g. proportional to the distance or shipping cost from facility j to client i). We introduce a binary decision variable y_i which is 1 if a facility is opened at site j, and 0 otherwise. We also have binary decision variables a_{ij} which equal 1 if demand point i is assigned to facility j, and 0 otherwise. One formulation is:

$$\boldsymbol{min} \sum_{\boldsymbol{j} \in \boldsymbol{j}} \boldsymbol{f_j}\ \boldsymbol{y_j} + \sum_{\boldsymbol{i} \in \boldsymbol{I}} \sum_{\boldsymbol{j} \in \boldsymbol{J}} \boldsymbol{c_{ij}}\ \boldsymbol{a_{ij}}$$

$$s.t. \quad \sum_{j \in J} \boldsymbol{a_{ij}} = 1, \quad \forall i \in I,$$

$$a_{ij} \leq y_j, \quad \forall i \in I, \qquad \forall j \in J,$$

$$y_j \in \{0,1\}, \qquad a_{ij} \in \{0,1\}, \ \forall i \in I, \qquad \forall j \in J.$$

Here, the first constraint means every demand point iii is assigned to exactly one facility (no customer is left unserved and no customer is served by two facilities). The second constraint ensures that a demand point can only be assigned to facility j if that facility is open ($y_j = 1$). The objective function combines the total fixed costs of opened facilities with the total transportation (service) costs for all assignments. This formulation assumes facilities have unlimited capacity; in a capacitated version, one would add constraints to limit the total demand assigned to each facility.

Location-allocation problems are generally NP-hard (except in special cases), so they are challenging to solve exactly for large-scale instances. However, modern integer programming solvers can handle moderately sized instances (on the order of tens of potential facilities and hundreds of demand points) to optimality. For larger instances or more complex variants, heuristic and metaheuristic algorithms are widely used in practice. Techniques such as greedy add/drop heuristics, genetic algorithms, and simulated annealing have been successfully applied to large L-A problems, including those arising in logistics network design.

Example 1 **(Automotive Distribution Center Location):** A real-world illustration of a facility location decision in the Slovak automotive sector is the establishment of BMW Group's regional distribution center for spare parts near Bratislava. Although BMW does not produce cars in Slovakia, in 2016 it opened a 25,000 m² logistics hub at Prologis Park Bratislava to serve Central and Eastern Europe. The choice of location was driven by proximity to core transport corridors and markets – the site lies on the Pan-European Corridor IV highway, which was a key factor in BMW's decision [7]. By situating this parts warehouse in western Slovakia, BMW can quickly dispatch parts to 12 surrounding countries within its Central European service region [7]. This location-allocation solution minimizes transit times to dealerships and cuts costs by centralizing inventory in a logistically advantageous spot. In terms of our model, the "facility" (the warehouse) was chosen among candidate locations for its minimal total distribution cost to the region's demand points. The success of this hub underscores the importance of rigorous location analysis: it provides a high service-level network for aftersales logistics while keeping transportation expenses in check.

Example 2 **(Automotive Plant Location Decision):** Another example highlighting location-allocation considerations is Jaguar Land Rover's decision to build a new manufacturing plant in Nitra, Slovakia. In 2015, after examining various sites worldwide, JLR selected Nitra as the preferred location for its €1.4 billion factory. A major reason cited was the strong existing supplier base and logistics infrastructure in Western Slovakia [6]. In terms of location theory, JLR essentially solved a complex location-allocation problem: it chose an optimal site that would allow it to *allocate* its incoming supply chain efficiently (many of JLR's key suppliers are within easy reach of Nitra, reducing inbound transport costs) and to distribute finished vehicles efficiently to important markets. This was facilitated by Slovakia's well-integrated road and rail network, with a robust transportation infrastructure to handle exports (approximately half of Slovakia's exported cars are moved by rail) [1]. The Nitra plant began production in 2018, and its performance has validated the location choice – the facility is well-served by highways and rail, and it leverages the country's dense cluster of automotive suppliers. This example demonstrates how strategic factory location is intertwined with transportation considerations: the goal was to minimize overall logistics costs (including both the inbound shipment of parts and the outbound distribution of vehicles) while maintaining high service levels. In effect, JLR's investment decision was guided by the same principles that underpin formal location-allocation models, balancing fixed facility costs with transportation efficiencies across its supply chain network.

Integrating Transportation and Location Decisions

In practice, transportation and location-allocation decisions are often interdependent. Businesses must frequently make facility location choices (where to site a plant, warehouse, or distribution hub) in tandem with transportation planning (how to route deliveries or shipments), as part of an overall supply chain *network design*. Solving this combined problem optimally is complex, but even partial integration can yield significant benefits. For example, a company might decide to open a slightly more expensive warehouse in a central location if it substantially lowers transport costs to customers, resulting in a net gain. Advanced optimization models and software now allow companies to tackle such integrated network design problems using a combination of exact methods and heuristics.

The theoretical frameworks outlined above – from the linear programming solution of the transportation problem to the integer programming formulations of facility location – provide essential decision-support tools. In Slovakia's automotive industry, these tools underpin decisions such as designing milk-run collection circuits, locating regional logistics centers, and expanding supplier parks near assembly plants. As the industry evolves (e.g. with the shift to electric vehicles and new distribution models), transport and logistics systems theory will remain indispensable. Companies and researchers will continue to refine these models to incorporate new factors (like carbon emissions or risk mitigation) and to solve ever-larger instances with modern computational techniques. The end result is a more efficient and resilient logistics network, which is crucial for competitiveness in today's fast-changing market.

Conclusion

Efficient transport and logistics systems are the backbone of supply chain competitiveness. The transportation and location-allocation problems discussed in this article form the theoretical core for optimizing these systems. The transportation problem offers a powerful yet tractable model for minimizing distribution costs between fixed supply and demand nodes, whereas location-allocation models guide strategic placement of facilities to balance cost, service level, and other criteria. The current research landscape shows a maturing of solution techniques for these problems, with an emphasis on integration (combining multiple decision layers), handling uncertainty, and addressing sustainability considerations. Real-world examples from Slovakia's automotive logistics sector illustrate the tangible impact of these theoretical models: from daily operations like inbound parts delivery to high-level investments like new factories and distribution hubs, optimization principles drive better outcomes. By applying the theory of transport and logistics systems, organizations can achieve significant cost savings, service improvements, and greater resilience – outcomes that are vitally important in a global supply chain environment characterized by demand volatility and constant pressure for efficiency.

References

[1] Mordor Intelligence. *Slovakia Freight and Logistics Market – Growth, Trends, Forecast (2025–2030)*. Industry Report, 2025. (Accessed online on 02 May 2024).

 DOI: 10.26552/pte.C.2024.1.4 ISSN 1336-8281

[2] Kádárová, Jaroslava – Trebuňa, Peter – Lachvajderová, Laura. *Model for Optimizing the Ratios of the Company Suppliers in Slovak Automotive Industry*. Sustainability, 2021, 13(21): 11597. DOI: 10.3390/su132111597

[3] Liptáková, Jana. *Slovakia beats record in car production, again* [online]. The Slovak Spectator, 13 January 2020 [cit. 2024-04-02]. Available from: https://spectator.sme.sk/c/22301073/slovakia-beats-record-in-car-production-again.html

[4] Song, Yongjie. *The Classic Transportation Problem*. The Geographic Information Science & Technology Body of Knowledge (3rd Quarter 2021 Edition) [online], 2021. DOI: 10.22224/gistbok/2021.3.5

[5] Vafaeinejad, Alireza – Bolouri, Samira – Alesheikh, Ali A. – Panahi, Mahdi – Lee, Chang-Wook. *The Capacitated Location-Allocation Problem Using the VAOMP (Vector Assignment Ordered Median Problem) Unified Approach in GIS*. Applied Sciences, 2020, 10(23): 8505. DOI: 10.3390/app10238505

[6] LOGISTIK express. *Good logistics infrastructure makes Slovakia attractive for Jaguar Land Rover* [online]. Press release, 12 August 2015 [cit. 2024-04-02]. Available from: https://www.logistik-express.com/good-logistics-infrastructure-makes-slovakia-attractive-for-jaguar-land-rover/

[7] Prologis. *Build-to-Suit for BMW in Bratislava, Slovakia* [online]. 2016 [cit. 2024-04-02]. Available from: https://www.prologis.com/insights/success-stories/build-suit-bmw-bratislava-slovakia

[8] Stankovský, Peter – Cibulka, Viliam. Present situation of integrated logistics in Slovak automotive industry. In: *Annals of DAAAM for 2011 & Proceedings of the 22nd International DAAAM Symposium*. Vienna: DAAAM International, 2011, pp. 1261–1263.